

# Constraining cosmologies with fundamental constants I. Quintessence and K-Essence

Rodger I. Thompson<sup>1\*</sup>, C.J.A.P. Martins<sup>2,3</sup> and P.E. Vielzeuf<sup>2,4,5</sup>,

<sup>1</sup>*Steward Observatory, University of Arizona, Tucson, AZ 85721, USA*

<sup>2</sup>*Centro de Astrofísica, Universidade do Porto, Rua das Estrelas, 4150-762 Porto, Portugal*

<sup>3</sup>*Department of Applied Mathematics and Theoretical Physics, Centre for Mathematical Sciences, University of Cambridge, Wilberforce Road, Cambridge CB3 0WA, United Kingdom*

<sup>4</sup>*Faculdade de Ciencias, Universidade do Porto, Rua do Campo Alegre 687, 4169-007 Porto, Portugal*

<sup>5</sup>*Université Paul Sabatier-Toulouse III, 118 Route de Narbonne 31062 Toulouse Cedex 9, France*

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## ABSTRACT

Many cosmological models invoke rolling scalar fields to account for the observed acceleration of the expansion of the universe. These theories generally include a potential  $V(\phi)$  which is a function of the scalar field  $\phi$ . Although  $V(\phi)$  can be represented by a very diverse set of functions, recent work has shown that under some conditions, such as the slow roll conditions, the equation of state parameter  $w$  is either independent of the form of  $V(\phi)$  or is part of a family of solutions with only a few parameters. In realistic models of this type the scalar field couples to other sectors of the model leading to possibly observable changes in the fundamental constants such as the fine structure constant  $\alpha$  and the proton to electron mass ratio  $\mu$ . Although the current situation on a possible variance of  $\alpha$  is complicated there are firm limitations on the variance of  $\mu$  in the early universe. This paper explores the limits this puts on the validity of various cosmologies that invoke rolling scalar fields. We find that the limit on the variation of  $\mu$  puts significant constraints on the product of a cosmological parameter  $w + 1$  times a new physics parameter  $\zeta_\mu^2$ , the coupling constant between  $\mu$  and the rolling scalar field. Even when the cosmologies are restricted to very slow roll conditions either the value of  $\zeta_\mu$  must be at the lower end of or less than its expected values or the value of  $w + 1$  must be restricted to values vanishingly close to 0. This implies that either the rolling scalar field is very weakly coupled with the electromagnetic field, small  $\zeta_\mu$ , very weakly coupled with gravity,  $(w + 1) \approx 0$  or both. These results stress that adherence to the measured invariance in  $\mu$  is a very significant test of the validity of any proposed cosmology and any new physics it requires. The limits on the variation of  $\mu$  also produces a significant tension with the reported changes in the value of  $\alpha$ .

**Key words:** (cosmology:) cosmological parameters – dark energy – theory – early universe .

## 1 INTRODUCTION

Tracing the values of the fundamental constants through the history of the universe provides strong constraints on the possibility of cosmologies other than the standard  $\Lambda$ CDM universe and new physics that deviates from the standard model. In this investigation we use the observed limits on the variation of the proton to electron mass ratio  $\mu$  as a new

input parameter for three quintessence cosmologies and K-Essence. Each of these cosmologies postulates a rolling scalar field  $\phi$  with a potential  $V(\phi)$ . Realistic models of this class expect the scalar field to also have non-zero couplings to sectors other than gravity unless an unknown symmetry is postulated to suppress them (Carroll 1998). Here we assume the simplest non-vanishing coupling to the electromagnetic sector and a unification scenario of the type described in Coc et al. (2007) that leads to a change in  $\mu$  that is related to a change in  $\alpha$ .

\* E-mail: rit@email.arizona.edu  
Carlos.Martins@astro.up.pt  
up110370652@alunos.fc.up.pt (PEV)

(RIT);  
(CJAPM);

At the epoch of each  $\frac{\Delta\mu}{\mu}$  measurement there is a constraint placed on the product of the coupling of  $\mu$  to

Object	Redshift	$\Delta\mu/\mu$	error	$(w+1)\zeta_\mu^2$	Accuracy	Reference
Q0347-383	3.0249	$2.1 \times 10^{-6}$	$\pm 6. \times 10^{-6}$	$\leq 3.8 \times 10^{-11}$	$1\sigma$	Wendt & Reimers (2008)
Q0405-443	2.5974	$10.1 \times 10^{-6}$	$\pm 6.2 \times 10^{-6}$	$\leq 4.0 \times 10^{-11}$	$1\sigma$	King et al. (2009)
Q0528-250	2.811	$3.0 \times 10^{-7}$	$\pm 3.7 \times 10^{-6}$	$\leq 1.4 \times 10^{-11}$	$1\sigma$	King et al. (2011)
J2123-005	2.059	$5.6 \times 10^{-6}$	$\pm 6.2 \times 10^{-6}$	$\leq 4.0 \times 10^{-11}$	$1\sigma$	Malec et al. (2010)
PKS1830-211	0.89	0.0	$\pm 6.3 \times 10^{-7}$	$\leq 6.5 \times 10^{-13}$	$3\sigma$	Ellingsen, Voronkov, Breen & Lovell (2012)
B0218+357	0.6847	0.0	$\pm 3.6 \times 10^{-7}$	$\leq 2.8 \times 10^{-13}$	$3\sigma$	Kanekar (2011)

**Table 1.** Observational constraints used in this analysis.

the rolling scalar field and the equation of state parameter  $w$  that is independent of the cosmology except for the form of the equation describing the dark energy density  $\Omega_\phi(z)$ . The evolution of  $\mu$  and  $w$  within those constraints, however, is dependent on the particular cosmology. We investigate one freezing cosmology, slow roll quintessence, and 3 thawing cosmologies, hilltop quintessence, non-minimal quintessence and K-Essence. Freezing models start with the equation of state parameter  $w$  different from  $-1$  at early times and approaching  $-1$  at the present time while thawing models start with  $w$  close to  $-1$  at early times and deviate from  $-1$  at the present epoch. Each of these cosmologies has the advantage of having a family of solutions which is a function of a small number of parameters (Dutta & Scherrer (2011), Gupta, Saridakis & Sen (2009), Chiba, Dutta & Scherrer (2009), Dutta & Scherrer (2008)) We will follow the methodology laid out in Thompson (2012) for just slow roll quintessence to investigate the constraints on all four cosmologies.

## 2 OBSERVATIONAL CONSTRAINTS

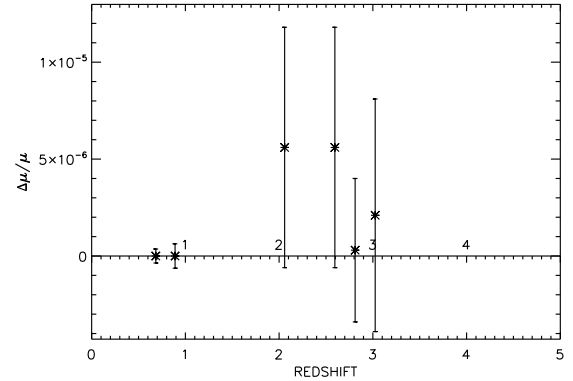
Appendix A gives a comprehensive list of measurements of  $\mu$  using astronomical observations. Based on this list Table 1 gives the most constraining limits on the value of  $\Delta\mu/\mu$ . The listings for radio observations of PKS1830-211 and B0218+357 give the  $3\sigma$  limits about a null result. All of the other observations contain the null result in their  $1\sigma$  limits. Figure 1 shows the errors from Table 1 plotted as a function of redshift. These are the measurements used in establishing the constraints used in this analysis.

## 3 VARYING $\mu$ IN THE CONTEXT OF NEW PHYSICS

A time varying value of  $\mu$  is not allowed in the Standard Model so any variation in  $\mu$  introduces new physics. As in Thompson (2012) we follow the discussion of Nunes & Lidsey (2004), hereinafter NL, which actually discusses varying values of the fine structure constant  $\alpha$ . The same physics applies to  $\mu$  with the two constants connected by

$$\frac{\dot{\mu}}{\mu} \sim \frac{\dot{\Lambda}_{QCD}}{\Lambda_{QCD}} - \frac{\dot{\nu}}{\nu} \sim R \frac{\dot{\alpha}}{\alpha} \quad (1)$$

given in Avelino et al. (2006). In equation 1  $\Lambda_{QCD}$  is the QCD scale,  $\nu$  is the Higgs vacuum expectation value and  $R$  is a scalar often considered to be on the order of  $-40$  to  $-50$  (Avelino et al. 2006). In the rest of the discussion we assume

**Figure 1.** The observed values of  $\Delta\mu/\mu$  and their associated errors from Table 1. Note that the two lowest redshift errors are  $3\sigma$  errors while the rest are  $1\sigma$  error bars.

the value of  $R$  to be  $-40$  but consider possible variations from this value in a later section on the tension between the limits on the variation of  $\mu$  and the reported variation of  $\alpha$ . NL consider the simplest possible coupling of  $\mu$  with a rolling scalar field  $\phi$ , namely a linear coupling given by

$$\frac{\Delta\mu}{\mu} = R\zeta_\alpha\kappa(\phi - \phi_0) = \zeta_\mu\kappa(\phi - \phi_0) \quad (2)$$

where  $\zeta_x$  ( $x = \alpha, \mu$ ) is the coupling constant,  $\kappa = \frac{\sqrt{8\pi}}{m_p}$  and  $m_p$  is the Planck mass. The coupling constants  $\zeta_x$  are considered constant in time. Certainly other forms of coupling can be considered but in the absence of any information on its nature we choose to use the simplest form. Further, since it is known by observation that any variation of  $\mu$  is small, a linear coupling approximation is legitimate at least out to redshifts on the order of 4. The rolling potential  $V(\phi)$  is a function of the scalar  $\phi$  and the equation of state  $w$  is given by

$$w \equiv \frac{p_\phi}{\rho_\phi} = \frac{\dot{\phi}^2 - 2V(\phi)}{\dot{\phi}^2 + 2V(\phi)} \quad (3)$$

(NL) then show that  $w+1$  is also given by

$$w+1 = \frac{(\kappa\phi')^2}{3\Omega_\phi} \quad (4)$$

where  $\Omega_\phi$  is the dark energy density. Here  $\dot{\phi}$  and  $\phi'$  indicate differentiation with respect to cosmic time and to  $N = \log a$  respectively where  $a$  is the scale factor of the universe. Equation 2 shows that

$$\phi' = \frac{\mu'}{\kappa\zeta_\mu\mu} \quad (5)$$

It follows from 4 and 5 that

$$w + 1 = \frac{(\mu'/\mu)^2}{3\zeta_\mu^2\Omega_\phi} = \frac{(\alpha'/\alpha)^2}{3\zeta_\alpha^2\Omega_\phi} \quad (6)$$

which establishes a connection between the evolution of  $w$  and  $\mu$ . Note that for the phantom case,  $w < -1$ , the two right hand terms in equation 6 are preceded by a minus sign.

Since  $\mu' = a(\frac{d\mu}{da})$  we can find the variance of  $\mu$  relative to its present day value at any scale factor  $a$  by performing the integral

$$\frac{\Delta\mu}{\mu} = \zeta_\mu \int_1^a \sqrt{3\Omega_\phi(x)(w(x)+1)} x^{-1} dx \quad (7)$$

The value of  $w+1$  versus redshift or scale factor is set by the different cosmologies. For phantom cosmologies the factor of  $(w+1)$  in equation 7 is replaced by  $-(w+1)$ .

#### 4 CONSTRAINTS THAT ARE RELATIVELY INDEPENDENT OF THE COSMOLOGICAL MODEL

Equation 6 provides a constraint on the combination of a cosmological parameter  $w$  and a new physics parameter  $\zeta_\mu$  relative to the limits on  $\mu'/\mu$

$$(w+1)\zeta_\mu^2 = \frac{(\mu'/\mu)^2}{3\Omega_\phi} \quad (8)$$

that is independent of the form of the potential  $V(\phi)$ . Again utilizing that  $\mu' = a(\frac{d\mu}{da})$  we can write

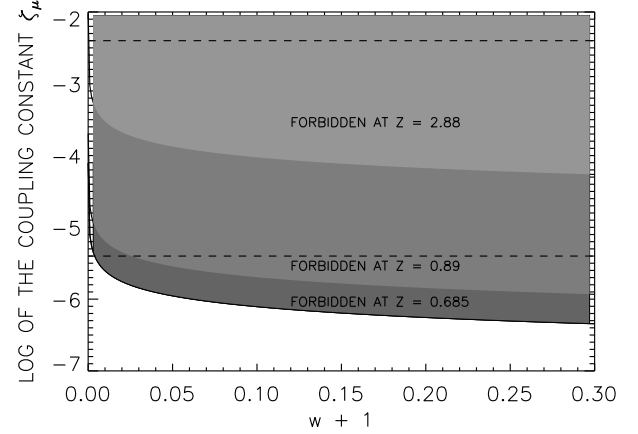
$$(w+1)\zeta_\mu^2 = \frac{(\Delta\mu/\mu)^2(a/\Delta a)^2}{3\Omega_\phi} \quad (9)$$

giving the constraint on  $(w+1)\zeta_\mu^2$  as a function of  $\Delta\mu$  and the dark energy density  $\Omega_\phi$ . Any combination of a given cosmology and value of  $\zeta_\mu$  must satisfy the constraint given by equation 9 at the redshift of the observation. Different cosmologies, however, take separate paths in the  $\Delta\mu$  redshift plane to meet the constraints. In order to proceed we now impose the condition that the dark energy density factor  $\Omega_\phi$  is given by.

$$\Omega_\phi = [1 + (\Omega_{\phi 0}^{-1} - 1)a^{-3}]^{-1} \quad (10)$$

In equation 10 the subscript 0 refers to the present day value. This form for  $\Omega_\phi$  assumes that  $w$  is close to  $-1$  so that the  $e^{-3 \int \frac{(1+w(z))dz}{1+z}}$  term that multiplies  $a^{-3}$  in the full expression is approximately 1. This applies for the cases considered in this work. An examination of the exact dark energy density solutions for each of the cosmologies indicates that most variations from equation 10 are less than 10% at redshifts less than 4 with the maximum being 20% for some K-Essence cases. Figure 2 is therefore a good representation of the forbidden parameter space. The bounds on  $(w+1)\zeta_\mu^2$  at each epoch are listed in Table 1.

Another way to look at the constraints imposed by the  $\Delta\mu$  limits is to look at the allowed and forbidden areas in the  $\zeta_\mu$  ( $w+1$ ) plane as a function of redshift. Figure 2 shows the allowed and forbidden areas for the most restrictive low redshift constraints, B0218+357 at  $z = 0.6847$  (Kanekar 2011) and PKS 1830-211 at  $z = 0.89$  (Ellingsen, Voronkov, Breen & Lovell 2012) as well as the most restrictive high redshift constraint, Q0528-250 at  $z =$



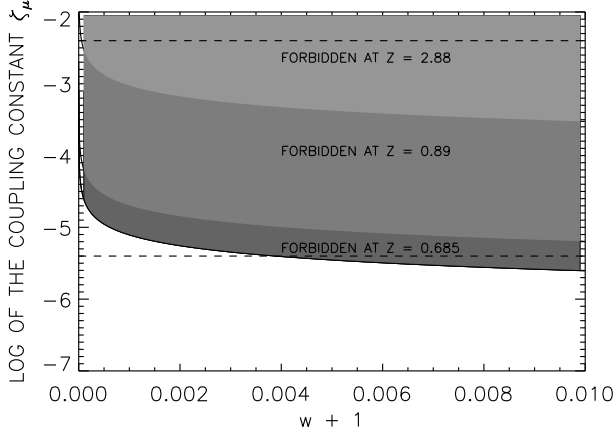
**Figure 2.** The figure shows the forbidden and allowed parameter space in the  $\zeta_\mu$ , ( $w+1$ ) plane based on the three most restrictive low and high redshift observations. The upper light shaded area is for the constraint at a redshift of 2.811, the middle darker area and above are for a redshift of 0.89, and lower dark shaded area and above is for the constraint at a redshift of 0.685. The dashed lines indicate the upper and lower most likely limits on the coupling factor  $\zeta_\mu$  as discussed in the text.

2.811 (King et al. 2011). In the figure all of the space above the solid lines is forbidden. Figure 2 is a fundamental result of this paper. It applies to all cosmologies for which equation 10 for the dark energy density is valid. First shown in a slightly different format in Thompson (2012), it defines the allowed parameter space for  $w$  and  $\zeta_\mu$ . All cosmologies must adhere to the allowed space at the redshifts of the observations. Different cosmologies take different paths through the allowed parameter space, therefore, filling in the diagram with measurements at a large number of redshifts with greatly improved accuracy is an important task.

NL use the work of Copeland et al. (2004) to set likely bounds on the value of  $\zeta_\alpha$  of  $\zeta_\alpha \sim 10^{-7} - 10^{-4}$ . For a R value of -40 this translates to a likely range for  $\zeta_\mu$  of  $\zeta_\mu \sim -4 \times 10^{-6}$  to  $-4 \times 10^{-3}$ . These bounds are shown by the dashed horizontal lines figure 2. Figure 3 is a greatly magnified view of the  $w+1$  space from 0 - 0.01 which shows that at the lowest expected value of  $\zeta_\mu$  only the space with  $(w+1) < 0.004$  is allowed at a redshift of 0.685. Coupling constants near the high end of the expected value require  $(w+1)$  to be essentially zero. However, as discussed later, setting  $\zeta_\mu$  to less than  $5 \times 10^{-7}$  allows a full range of  $(w+1)$  values. Inclusion of the two radio observations at redshifts of 0.685 and 0.89 results in a much more restricted parameter space than presented in Thompson (2012) that only included the results from optical observations of H<sub>2</sub>. We next investigate how figure 2 impacts the the allowed parameters for the four cosmologies.

#### 5 THE PARAMETERIZED SOLUTIONS

Each of the four cosmologies, examined in this work have parameterized solutions for the value of  $w+1$  as a function of scale factor or redshift. As shown in Thompson (2012) this also leads to solutions for  $\Delta\mu/\mu$  through Equation 6. In



**Figure 3.** The figure gives a detailed view of the narrow allowed limits on  $w + 1$  if the value of  $\zeta_\mu$  is taken at its lower expected limit shown by the lower dashed line. At a redshift of 0.685  $w + 1$  is constrained to be less than 0.004 unless the coupling constant is reduced below its lowest expected value.

this section we examine the parameterized solutions for each of the four cosmologies and the subsequent solutions for the variance of  $\mu$ . In each case we use Geometrized units where  $8\pi G = 1$ . Once the parameterized solutions are established reasonable parameters are selected to provide test cases for each cosmology. In section 6 the value of  $\zeta_\mu$  is then adjusted to satisfy the constraints on  $\Delta\mu$  given in Table 1.

### 5.1 Slow Roll Conditions

In each of these cosmologies, except for hilltop quintessence, we impose the standard slow roll conditions on the potential  $V(\phi)$ .

$$\lambda^2 \equiv \left(\frac{1}{V} \frac{dV}{d\phi}\right)^2 \ll 1 \quad (11)$$

$$\left|\frac{1}{V} \frac{d^2V}{d\phi^2}\right| \ll 1 \quad (12)$$

These conditions produce a very flat potential and are generally the same conditions for a minimal variation in  $\mu$ . This means that the restrictions on the parameter space for non-slow roll cosmologies would probably be even stricter than in the slow roll case. In many cases, such as slow roll quintessence the value of  $\lambda$  in Equation 11 is taken to be a constant value equal to  $\lambda_0$  which then becomes one of the parameters.

#### 5.1.1 Slow Roll Quintessence

This cosmology was already treated in Thompson (2012) but we include it here for completeness. The dynamical equation is given by

$$\ddot{\phi} + 3H\dot{\phi} + \frac{dV}{d\phi} = 0 \quad (13)$$

and the parameterized solution for  $w + 1$  is given by Dutta & Scherrer (2011) as

$$1 + w = \frac{1}{3}\lambda_0^2 \left[ \frac{1}{\sqrt{\Omega_\phi}} - \left( \frac{1}{\Omega_\phi} - 1 \right) (\tanh^{-1}(\sqrt{\Omega_\phi}) + C) \right]^2 \quad (14)$$

The parameter  $C$  characterizes the family of solutions and is set by and early condition on  $w = w_i$ .

$$C = \pm \frac{\sqrt{3(1 + w_i)\Omega_{\phi_i}}}{\lambda_0} \quad (15)$$

$C$  is set by picking the value of  $w$  at some early epoch such as  $z = 5$  as we will do in a later section. This value  $w_i$  then sets the solution for  $w + 1$  and  $\Delta\mu/\mu$  at all other epochs. As shown in Thompson (2012) equation 6 and equation 10 give the evolution of  $\mu$  as

$$\begin{aligned} \frac{\Delta\mu}{\mu} = \zeta_\mu \lambda_0 \int_1^a \{ & 1 - [(1 + (\Omega_0^{-1} - 1)x^{-3})^{-1/2} \\ & - (1 + (\Omega_0^{-1} - 1)x^{-3})^{1/2}] \\ & \times [\tanh^{-1}(1 + (\Omega_0^{-1} - 1)x^{-3})^{1/2} + C] \} x^{-1} dx \end{aligned} \quad (16)$$

#### 5.1.2 Hilltop Quintessence

The dynamical equation for hilltop quintessence is the same as for slow roll quintessence. In hilltop quintessence the scalar field is rolling down a potential from a position very near the maximum of the potential. The cosmology adheres to the first slow roll condition but in some cases the second slow roll condition is relaxed. In this section we follow the discussion of Dutta & Scherrer (2008) in developing the parameterized solutions. Dutta & Scherrer (2008) show that  $w + 1$  is given by

$$\begin{aligned} 1 + w(a) = (1 + w_0)a^{3(K-1)} \\ \frac{[(F(a) + 1)^K(K - F(a)) + (F(a) - 1)^K(K + F(a))]^2}{[(\Omega_{\phi_0}^{-\frac{1}{2}} + 1)^K(K - \Omega_{\phi_0}^{-\frac{1}{2}}) + (\Omega_{\phi_0}^{-\frac{1}{2}} - 1)^K(K + \Omega_{\phi_0}^{-\frac{1}{2}})]^2} \end{aligned} \quad (17)$$

where  $F(a)$  is given by

$$F(a) = \sqrt{1 + (\Omega_{\phi_0}^{-1} - 1)a^{-3}} \quad (18)$$

The parameter  $K$  is given by

$$K = \sqrt{1 - (4/3)V''(\phi_*)/V(\phi_*)} \quad (19)$$

where  $\phi_*$  is the value of  $\phi$  at the maximum. At the maximum  $V''(\phi_*) < 0$  therefore  $K > 1$ . For true slow roll conditions  $K$  should not be much greater than 1.

The variance of  $\mu$  is then given by

$$\begin{aligned} \frac{\Delta\mu}{\mu} = \zeta_\mu \sqrt{1 + w_0} \int_1^a (x^{\frac{3(K-1)}{2}} \frac{\sqrt{3}}{\sqrt{1 + (\Omega_0^{-1} - 1)x^{-3}}}) \\ \frac{[(F(x) + 1)^K(K - F(x)) + (F(x) - 1)^K(K + F(x))]^2}{[(\Omega_{\phi_0}^{-\frac{1}{2}} + 1)^K(K - \Omega_{\phi_0}^{-\frac{1}{2}}) + (\Omega_{\phi_0}^{-\frac{1}{2}} - 1)^K(K + \Omega_{\phi_0}^{-\frac{1}{2}})]^2} x^{-1} dx \end{aligned} \quad (20)$$

#### 5.1.3 Non-Minimal Quintessence and Phantom

As the name implies, in non-minimal quintessence and phantom cosmologies the quintessence and phantom fields couple with gravity in a non-minimal way. We will follow the discussion of Gupta, Saridakis & Sen (2009) who introduce the usual parameter  $\epsilon$  which has a value of +1 for quintessence and -1 for phantom where the value of  $w$  is less than -1. The dynamical equation for non-minimal models is given by

$$\ddot{\phi} + 3H\dot{\phi} + 6\xi(\dot{H} + 2H^2)\phi + \epsilon V'(\phi) = 0 \quad (21)$$

In equation 21  $\xi$  is the non-minimal coupling parameter, usually set to  $1/6$ , which we will use here. The results are relatively insensitive to the value, however. Gupta, Saridakis & Sen (2009) show that the parameterized solution for the equation of state is given by

$$1 + w_\phi(a) = \epsilon \frac{1}{9} \left\{ \frac{[1 + (\Omega_{\phi 0}^{-1} - 1)a^{-3}](1 - \Omega_{\phi 0})}{1 + (a^3 - 1)\Omega_{\phi 0}} \right\}^{2 - (8\xi/3)} \\ \times \{6\epsilon\sqrt{2}z_0\xi B([1 + (\Omega_{\phi 0}^{-1} - 1)a^{-3}]^{-1}; \frac{1}{2} - \frac{4\xi}{3}, -1 + \frac{4\xi}{3}) \\ + [\sqrt{3}\lambda_0(1 - 2\xi) - 6\epsilon\sqrt{2}z_0\xi] \\ \times B([1 + (\Omega_{\phi 0}^{-1} - 1)a^{-3}]^{-1}; \frac{3}{2} - \frac{4\xi}{3}, -1 + \frac{4\xi}{3})\}^2 \quad (22)$$

where  $B$  is the incomplete Beta function.  $\lambda_0$  is the value of the first slow roll condition and again assumed to be constant.  $z_0$  is the average value of the auxiliary variable  $z = \frac{\kappa\phi}{\sqrt{6}}$ . Here  $\phi$  is the scalar field and  $\kappa^2 = 8\pi G$ . The nominal value of  $z_0$  is  $10^{-5}$  with the result again very insensitive to the value.

Equation 23 then gives the variation of  $\mu$  with scale factor  $a$  as

$$\frac{\Delta\mu}{\mu} = \frac{\zeta_\mu\epsilon}{\sqrt{3}} \int_1^a \frac{1}{\sqrt{1 + (\Omega_{\phi 0} - 1)x^{-3}}} \\ \left\{ \frac{[1 + (\Omega_{\phi 0}^{-1} - 1)x^{-3}](1 - \Omega_{\phi 0})}{1 + (x^3 - 1)\Omega_{\phi 0}} \right\}^{1 - (4\xi/3)} \\ \times \{6\epsilon\sqrt{2}z_0\xi B([1 + (\Omega_{\phi 0}^{-1} - 1)x^{-3}]^{-1}; \frac{1}{2} - \frac{4\xi}{3}, -1 + \frac{4\xi}{3}) \\ + [\sqrt{3}\lambda_0(1 - 2\xi) - 6\epsilon\sqrt{2}z_0\xi] \\ \times B([1 + (\Omega_{\phi 0}^{-1} - 1)x^{-3}]^{-1}; \frac{3}{2} - \frac{4\xi}{3}, -1 + \frac{4\xi}{3})\} x^{-1} dx \quad (23)$$

Note that the flip in sign in the right hand part of equation 6 cancels the  $\epsilon = -1$  leading equation 23 making the phantom solutions for  $\Delta\mu/\mu$  indistinguishable from the quintessence solutions.

#### 5.1.4 K-Essence

In this section we follow the development of thawing slow roll k-essence by Chiba, Dutta & Scherrer (2009). K-essence introduces a non-cannonical kinetic term into the Lagrangian  $F(X)$  such that the pressure is given by

$$p(\phi, X) = V(\phi)F(X) \quad (24)$$

where  $\phi$  and  $V(\phi)$  are again the rolling scalar field and the potential of the field.  $X$  is given by

$$X = -\nabla^\mu\phi\nabla_\mu\phi/2 \quad (25)$$

The K-Essence equation of motion is given by

$$\ddot{\phi} + 3c_s^2 H\dot{\phi} + c_s^2 \frac{2XF_X - F}{F_X} \frac{V'}{V} = 0 \quad (26)$$

where

$$c_s^2 = \frac{F_X}{2XF_{XX} + F_X} \quad (27)$$

$F_X$  and  $F_{XX}$  indicate single and double derivatives with respect to  $X$  and  $V'$  is the derivative of  $V$  with respect to

$\phi$ . The slow roll conditions are the same as given in equations 11 and 12

Chiba, Dutta & Scherrer (2009) show that the equation of state for slow roll k-essence can be parametrized in the following form.

$$1 + w(a) = (1 + w_0)a^{3(K-1)} \\ \left( \frac{(K - F(a))(F(a) + 1)^K + (K - F(a))(F(a) - 1)^K}{(K - \Omega_{\phi 0}^{-1/2})(\Omega_{\phi 0}^{-1/2} + 1)^K + (K + \Omega_{\phi 0}^{-1/2})(\Omega_{\phi 0}^{-1/2} - 1)^K} \right)^2 \quad (28)$$

where

$$K = \sqrt{1 - \frac{4}{3} \frac{V''(\phi_i)}{F_X(0)V(\phi_i)^2}} \quad (29)$$

and

$$F(a) = \sqrt{1 + (\Omega_{\phi 0}^{-1} - 1)a^{-3}} \quad (30)$$

where  $F(a)$  in equation 30 is not  $F(X)$ .  $\phi_i$  is an initial value of  $\phi$  and  $\Omega_{\phi 0}$  and  $w_0$  are the present day values of  $\Omega_\phi$  and  $w$ . For K-Essence the equation for  $\frac{\Delta\mu}{\mu}$  is

$$\frac{\Delta\mu}{\mu} = \zeta_\mu\sqrt{1 + w_0} \int_1^a \frac{\sqrt{3}x^3 \frac{K-1}{2}}{\sqrt{1 + (\Omega_{\phi 0}^{-1} - 1)x^{-3}}} \\ \frac{(K - F(x))(F(x) + 1)^K + (K - F(x))(F(x) - 1)^K x^{-1}}{(K - \Omega_{\phi 0}^{-1/2})(\Omega_{\phi 0}^{-1/2} + 1)^K + (K + \Omega_{\phi 0}^{-1/2})(\Omega_{\phi 0}^{-1/2} - 1)^K} dx \quad (31)$$

For phantom solutions the leading term  $\sqrt{1 + w_0}$  becomes  $\sqrt{-(1 + w_0)}$  and as in the phantom non-minimal case the phantom K-Essence solutions are indistinguishable from the quintessence solutions.

## 6 FITTING THE CONSTRAINTS

Now that the parameterized solutions have been presented we can see what the parameters need to be in order to satisfy the constraints presented in Figure 1. Even with the parameterized solutions there is an infinite number of cosmologies possible. To limit the field the solution space needs to be constrained. We choose to place the constraints on the allowed values of the equation of state parameter  $w$ .

### 6.1 Case Values for $w$

The appropriate case values for  $w$  will be different for the one freezing cosmology, slow roll quintessence, than for the three thawing cosmologies. For the thawing cosmologies we choose present epoch values of  $w$  of -0.99, -0.95 and -0.9 as being consistent with the slow roll conditions. For cosmologies allowing phantom solutions we choose the mirror solutions of -1.01, -1.05 and -1.1 as well. For the freezing slow roll quintessence we choose values of  $w$  at redshift 5 of -0.5, -0.75 and -0.9. In each cosmology we then adjust the parameters used in Section 5 to achieve the desired initial values of  $w$ .

The case values for  $w$  are meant to span the range appropriate to the slow roll conditions with values very close to -1 to values 0.1 deviant from -1 that start to strain the slow roll conditions. The cases for slow roll quintessence satisfy the conditions for the redshifts of the observations but

start to become deviant at significantly higher redshifts. Allowing a larger deviation from  $-1$  pushes the values of  $\zeta_\mu$  even lower, consistent with the constraints in figure 2.

## 6.2 Setting the Parameters

Having chosen either the present day or redshift 5 values of the equation of state parameter  $w$  for the cosmologies we then vary  $\zeta_\mu$  to satisfy the  $\Delta\mu/\mu$  constraints since the  $w(a)$  tracks are independent of  $\zeta_\mu$ . First, in order to show the effect of the parameters on the solutions in figure 4, we choose a single value for  $\zeta_\mu$  such that all solutions for the parameter suite for a given cosmology fit the constraints. In Table 2, however, we list the largest absolute value of  $\zeta_\mu$  that fits the constraints for each individual parameter set along with the values of the parameters.

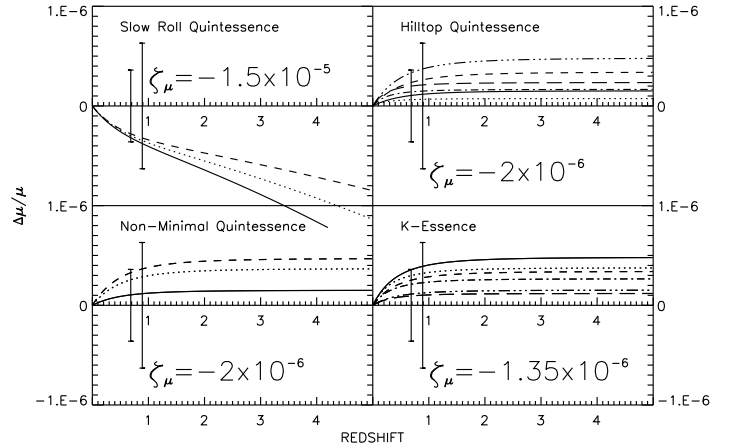
In the slow roll quintessence cosmology we only use negative values of the  $C$  parameter as they represent solutions to a field rolling down the potential. Positive values represent cases where the field initially rolls uphill (Dutta & Scherrer 2011). From Dutta & Scherrer (2011) we choose  $\lambda_0 = 0.08$  for this cosmology. Note that we could equally well have chosen to vary  $\lambda_0$  instead of  $\zeta_\mu$  to meet the constraints on slow roll cosmology, however that would have changed the value of the parameter  $C$  in equation 15. In hilltop quintessence we choose two values of  $K$ , 1.01 and 4.0 to represent a very slowly rolling solution for 1.01 and a solution,  $K = 4$ , in which the field rolls faster. In non-minimal quintessence we use the nominal values for  $\xi$  and  $z_0$  of  $1/6$  and  $10^{-5}$  given in Gupta, Saridakis & Sen (2009), however, as noted above the solutions are extremely insensitive to large changes in either of these parameters. The desired values of  $w$  in non-minimal quintessence are achieved by adjusting the slow roll parameter  $\lambda_0$ . The phantom solutions are produced by setting  $\epsilon$  to  $-1$  instead of  $+1$ . K-Essence also has phantom solutions. The  $K$  values for K-Essence are set by Equation 29 rather than by Equation 19 for the hilltop quintessence case. Since the potential is not starting at its maximum value the value of  $K$  can be less than 1. We bound the cases by letting  $K = 0.1, 2.0$ .

It is clear that even with the limited excursions of  $w$  from  $-1$  fitting the constraints requires the absolute values of  $\zeta_\mu$  in the lower range of expected values and in some cases lower than the lowest expected value of  $-4 \times 10^{-6}$ . Given the softness of the boundaries this result should probably taken as guidance in further calculations as opposed to invalidation of the concept. The results, however, are consistent with the Standard Model in which no variation in  $\mu$  is expected.

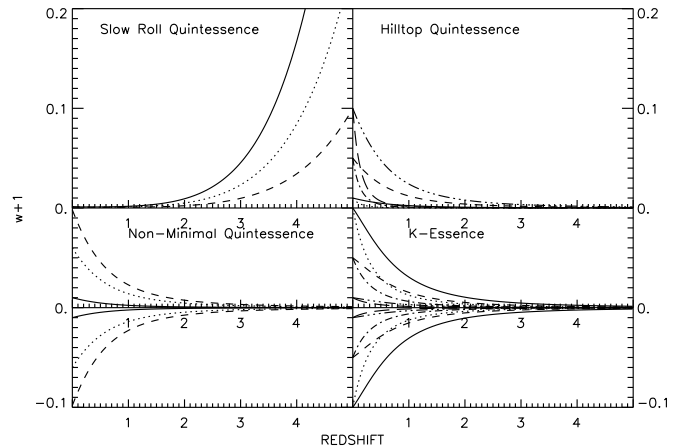
### 6.2.1 The Evolution of $w + 1$

Although each cosmology satisfies the same constraints the evolution of the equation of state  $w$  differs significantly. In particular the freezing slow roll quintessence cosmology can have  $w$  values significantly different than  $-1$  at early times and still satisfy the most restrictive constraint at redshift 0.6847. Figure 5 shows the evolution of the value of  $w + 1$  for each of the four cosmologies using the parameters shown in Table 2. The evolution of  $w$  is of course independent of the value of  $\zeta_\mu$ .

The 7th row of Table 2 lists the value of  $w + 1$  at a



**Figure 4.** This figure plots the evolution of  $\Delta\mu/\mu$  versus redshift for each of the four cosmologies. The value of  $\zeta_\mu$  has been adjusted in each cosmology so that all cases for that cosmology fall within the observational constraints. The higher redshift constraint at  $z=2.811$  is not plotted since it is larger than the plot size. The value of  $\zeta_\mu$  is marked in the lower left of each plot. Refer to Table 2 for the line style for each case. NB The value of  $\zeta_\mu$  in the figure is the value shown in the figure not the values in Table 2.



**Figure 5.** The figure shows the evolution of the equation of state parameter  $w$  by plotting the value of  $w + 1$  as a function of redshift for each of the four cosmologies. The last column of Table 2 contains the line style code for each of the cases.

redshift of 0.6847 for each cosmology solution. All of these values are quite low corresponding to the allowed  $w + 1$  space in Figure 2. Slow roll quintessence was able to satisfy the  $\Delta\mu/\mu$  constraints with higher values of  $\zeta_\mu$  than the other cosmologies and therefore has corresponding smaller deviations of  $w + 1$  from 0 as required by figure 2. This cosmology predicts very little deviation from  $w + 1 = 0$  out to a redshift of 2, the redshift region expected to be probed by currently proposed dark energy space missions. Cosmologies such as K-Essence that require very low absolute values of  $\zeta_\mu$  are able to achieve more significant deviations of  $w + 1$  from 0.

Cosmology	$\zeta_\mu$	$w^a$	$C$	$K$	$\lambda_0$	$(w+1)_{0.685}^b$	linestyle
Slow Roll Quintessence	$-1.69 \times 10^{-5}$	-0.5	-0.163611	-	0.08	0.0012	solid
	$-1.81 \times 10^{-5}$	-0.75	-0.11569	-	0.08	0.00090	dotted
	$-1.94 \times 10^{-5}$	-0.9	-0.073169	-	0.08	0.00067	dash
Hilltop Quintessence	$-6.85 \times 10^{-6}$	-0.99	-	1.01	-	0.00037	solid
	$-1.14 \times 10^{-5}$	-0.99	-	4.0	-	0.00036	dotted
	$-3.06 \times 10^{-6}$	-0.95	-	1.01	-	0.018	dash
	$-5.10 \times 10^{-6}$	-0.95	-	4.0	-	0.0018	dash dot
	$-2.16 \times 10^{-6}$	-0.9	-	1.01	-	0.037	dash 3dot
	$-3.61 \times 10^{-6}$	-0.9	-	4.0	-	0.0036	long dash
Non-Minimal Quintessence	$-6.88 \times 10^{-6}$	-0.99	-	-	0.32	0.0036	solid
	$-2.81 \times 10^{-6}$	-0.95	-	-	0.782	0.021	dotted
	$-2.20 \times 10^{-6}$	-0.9	-	-	1.0	0.035	dash
	$6.88 \times 10^{-6}$	-1.01	-	-	0.32	-0.0036	solid
	$2.81 \times 10^{-6}$	-1.05	-	-	0.782	-0.021	dotted
	$2.20 \times 10^{-6}$	-1.1	-	-	1.0	-0.035	dash
K-Essence	$-1.36 \times 10^{-6}$	-1.1	-	0.1	-	-0.045	solid
	$-1.63 \times 10^{-6}$	-1.1	-	2.0	-	-0.021	dotted
	$-1.93 \times 10^{-6}$	-1.05	-	0.1	-	-0.023	dash
	$-2.31 \times 10^{-6}$	-1.05	-	2.0	-	-0.010	dash dot
	$-4.31 \times 10^{-6}$	-1.01	-	0.1	-	-0.0046	dash 3dot
	$-5.16 \times 10^{-6}$	-1.01	-	2.0	-	-0.00021	long dash
	$-4.31 \times 10^{-6}$	-0.99	-	0.1	-	0.0045	long dash
	$-5.16 \times 10^{-6}$	-0.99	-	2.0	-	0.0021	dash 3dot
	$-1.93 \times 10^{-6}$	-0.95	-	0.1	-	0.023	dash dot
	$-2.31 \times 10^{-6}$	-0.95	-	2.0	-	0.010	dash
	$-1.36 \times 10^{-6}$	-0.90	-	0.1	-	0.045	dotted
	$-1.63 \times 10^{-6}$	-0.90	-	2.0	-	0.021	solid

**Table 2.** Observational constraints used in this analysis. The last column labeled linestyle indicates the linestyle used for that case in figure 4.

<sup>a</sup>  $w$  values for slow roll quintessence are for redshift  $z=5$ , all others are at redshift 0.

<sup>b</sup> The value of  $w+1$  at a redshift of 0.685

Imposing a value of  $\zeta_\mu$  less than  $3 \times 10^{-7}$  provides a large range of possible  $w$  values. If, however, the expected lower limit of  $\zeta_\mu = -4 \times 10^{-6}$  is imposed then the allowed deviation from  $w = -1$  at  $z = 0.6847$  is only about 0.004.

## 7 NEW PHYSICS IMPLICATIONS

Given the current constraints on  $\Delta\mu/\mu$  any significant deviation of  $w$  from  $-1$  requires a very low value of  $\zeta_\mu$  and a deviation greater than 0.004 requires a  $\zeta_\mu$  below the expected lower limit. The strong limits on the variance of  $\mu$  at redshifts below 1 require that the coupling of the scalar field with either or both of the gravitational and electromagnetic fields be very weak during that epoch.

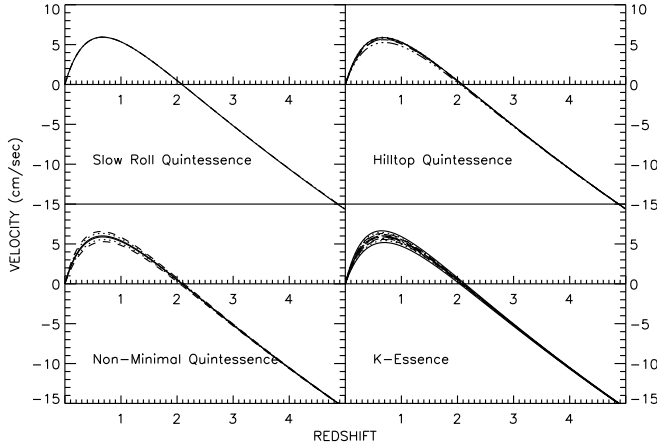
If we restrict ourselves to the assumptions we have used so far (slow-roll, a linear coupling, and a fixed value of  $R = -40$ ) then the spectroscopic bounds on  $\mu$  require the field to have a gravitational behavior almost exactly like that of a cosmological constant, even deep into the matter era. Given current theoretical expectations, such a behavior requires considerable fine-tuning (which is some ways is similar to the fine-tuning required to have a small but non-zero cosmological constant).

It is of course possible that not all of the above assumptions are correct. Slow-roll is observationally motivated at low redshifts (say  $z < 0.5$ ) when dark energy dominates

and the universe is accelerating, but need not hold at higher redshifts. For example, one could envisage situations where the field is moving relatively fast deep in the matter era, but then abruptly freezes at low redshift, perhaps due to a phase transition associated with the onset of dark energy domination. This scenario was briefly discussed (for the case of  $\alpha$ ), in Nunes, Dent, Martins & Robbers (2009).

Similarly, an assumption of linear coupling between the scalar field and the electromagnetic sector is a reasonable approximation at low redshifts but can conceivably break down at higher redshifts. The  $R$  parameter can have a different value, which points to a unification scenario that differs from the ones currently considered to be best motivated (Avelino et al. (2006), Coc et al. (2007)). For example, an  $R$  of order unity (in absolute value) suggests a scenario where unification occurs at relatively low energies, as is typically the case in models with large extra dimensions.

Finally, the coupling  $\zeta_\mu$  itself can be much smaller than anticipated. This degree of freedom is not independent from the others (in the context of the models being considered): given a certain non-zero level of  $\mu$  variation, a smaller coupling requires a faster moving field, and at some point the field must be moving so fast that the slow-roll approximation breaks down for such a field. In the limiting case the coupling can be exactly zero, and there would be no variation; however, as explained in Carroll (1998) this is again



**Figure 6.** The figure shows the Sandage Loeb Test velocities as a function of redshift for a twenty year baseline. Where resolvable, the line styles are the same as in the previous plots.

contrary to the simplest expectations for realistic models, as some unknown symmetry is needed to suppress the coupling.

Which of these scenarios is the correct one is not a question that our results can answer. However, our analysis highlights that the current results are at odds with our simplest expectations regarding scalar field models. At a more general level, this also highlights that null measurements can be extremely useful in constraining many theoretical scenarios.

## 8 SANDAGE LOEB TEST VALUES FOR THE FITTED COSMOLOGIES

In the era of large telescopes with the possibility of very high resolution spectrometers such as PEPSI and CODEX there has been discussion of direct measurements of the redshift drift due to the change in the expansion rate of the universe over time (Loeb 1998). This is generally called the Sandage Loeb Test. It has been recently considered as a method for measuring the dark energy component through the direct measurement of the drift in the redshift due to the accelerating expansion of the universe (Vielzeuf and Martins 2012). The change in velocity is given by

$$\Delta v = cH_0 t \left( 1 - \sqrt{(1+z)\Omega_m + \frac{(1+z)^{-2}\Omega_\phi}{e^{-3\int_0^z \frac{w(x)+1}{1+x} dx}} + \Omega_k} \right) \quad (32)$$

where  $\Omega_m$ ,  $\Omega_\phi$ , and  $\Omega_k$  are the ratio of the matter density, dark energy density and curvature density to the critical density and the equation of state evolution  $w(z)$  is dependent on the cosmology.

Figure 6 shows the Sandage Loeb Test velocity drifts for a twenty year baseline. All of the curves are very close to a  $\Lambda$ CDM signal, particularly for the slow roll quintessence cosmology. As pointed out by Vielzeuf and Martins (2012), a larger coupling factor leads to a slower moving scalar field and less deviation from the  $\Lambda$ CDM evolution. In contrast eg. to the results of Balbi & Quercellini (2007), the lack of significant dispersion in the Sandage Loeb Test curves is indicative of reduced parameter space resulting from the  $\Delta\mu/\mu$  constraints on the cosmologies considered here.

## 9 IMPLICATIONS ON VARYING $\alpha$

The observed invariance of  $\mu$  appears to be in tension with the reported temporal and spatial variance of  $\alpha$  (King et al. 2012). Although reported to be a spatial dipole we consider only the magnitude of the variance which is  $\Delta\alpha/\alpha = 1 \times 10^{-5}$  within the reported errors at an average redshift of 2 for the high redshift group. This compares with a conservative bound of  $\Delta\mu/\mu < 5 \times 10^{-6}$  for the same redshift from the observations referenced in this work. If both results are considered to be correct it requires that the value of  $R$  in equation 1 be 0.5 or less. This in turn requires that the values of  $\frac{\Lambda_{QCD}}{\Lambda_{QCD}}$  and  $\frac{\nu}{\nu}$  be very similar, contrary to generic GUTS models (Avelino et al. 2006). Another possibility is that only the Higgs VEV  $\nu$  changes and the quantum chromodynamic scale  $\Lambda_{QCD}$  is constant. Since the Higgs VEV scales all masses similarly to first order the ratio of the proton to electron mass remains unchanged while the Higgs VEV changes in  $\alpha$  would be observed. See, however, Coc et al. (2007) for a counter argument against varying one parameter and not the other. Barrow & Magueijo (2005) present the interesting opposite case of a constant  $\alpha$  with a varying  $\mu$ . If we entertain the possibility that the reported variation in  $\alpha$  is erroneous then neither constant has varied, consistent with a  $\Lambda$ CDM cosmology and the Standard Model of physics.

## 10 CONCLUSIONS

No variation in the value of  $\mu$  has been found to varying degrees of accuracy at six different redshifts between 0.685 and 3.02. This finding is consistent with either or both of  $\Lambda$ CDM cosmology and the Standard Model of Physics being valid. If, instead, the acceleration of the universe is due to a rolling scalar field that is both coupled to gravity and the electromagnetic field then one or both of the couplings has to be very weak as demonstrated by the very narrow allowed  $w+1$  space in figure 2 for any significant value of  $\zeta_\mu$ . Slow roll quintessence satisfies the invariance of  $\mu$  constraints with a low but reasonable  $\zeta_\mu$  value but with very minimal values of  $w+1$  from the present day out to redshifts of 2. The invariance of  $\mu$  is in tension with the reported variance of  $\alpha$  and requires a ratio of  $\Lambda_{QCD}$  change to  $\nu$  change much closer to 1 than expected. Given these conclusions the value of the fundamental constants as a function of redshift serves as a powerful constraint on new cosmologies and physics.

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## APPENDIX A: CURRENT DETERMINATIONS OF $\Delta\mu/\mu$

Table A lists the current determinations of  $\Delta\mu/\mu$  in distant galaxies and in the Milky Way. A subset of the most recent constraints are used in figures 1 and 2.

Object	Reference	Redshift	$\Delta\mu/\mu$
Q0347-383	Ivanchik, Rodriguez, Petitjean & Varshalovich (2002)	3.0249	$(5.7 \pm 3.8) \times 10^{-5}$
Q0347-383	Ivanchik, Rodriguez, Petitjean & Varshalovich (2002)	3.0249	$(12.5 \pm 4.5) \times 10^{-5}$
Q0347-383	Ivanchik, Petitjean, Rodriguez & Varshalovich (2003)	3.0249	$(\leq 8 \times 10^{-5})$
Q0347-383	Ubachs & Reinhold (2004)	3.0249	$(-0.5 \pm 3.6) \times 10^{-5}$
Q0347-383	Wendt & Reimers (2008)	3.0249	$(2.1 \pm 6) \times 10^{-6}$
Q0347-383	King et al. (2009)	3.0249	$(8.2 \pm 7.4) \times 10^{-6}$
Q0347-383	Thompson et al. (2009)	3.0249	$(-2.8 \pm 1.6) \times 10^{-5}$
Q0347-383	Wendt & Molaro (2011)	3.0249	$(1.5 \pm 1.1) \times 10^{-5}$
Q0347-383	Wendt & Molaro (2012)	3.0249	$(4.3 \pm 7.2) \times 10^{-6}$
347 & 405	Ivanchik et al. (2005)	comb	$(1.64 \pm 0.74) \times 10^{-5}$
347 & 405	Reinhold et al. (2006)	comb	$(2.46 \pm 0.6) \times 10^{-5}$
347 & 405	Ubachs et al. (2007)	comb	$(2.45 \pm 0.59) \times 10^{-5}$
Q0405-443	Thompson et al. (2009)	2.5974	$(3.7 \pm 14) \times 10^{-6}$
Q0405-443	King et al. (2009)	2.5974	$(10.1 \pm 6.2) \times 10^{-6}$
Q0528-250	Foltz, Chaffee & Black (1988)	2.811	$\leq 2.4 \times 10^{-4}$
Q0528-250	Cowie & Songaila (1995)	2.811	$\leq 7.0 \times 10^{-4}$
Q0528-250	Potekhin et al. (1998)	2.811	$\leq 2.0 \times 10^{-4}$
Q0528-250	King et al. (2009)	2.811	$(1.4 \pm 3.9) \times 10^{-6}$
Q0528-250	King et al. (2011)	2.811	$(0.3 \pm 3.7) \times 10^{-6}$
J2123-005	Malec et al. (2010)	2.059	$5.6 \pm 6.2) \times 10^{-6}$
PKS 1830-211	Henkel et al. (2009)	0.89	$\leq 1. \times 10^{-6}$
PKS 1830-211	Muller et al. (2011)	0.89	$\leq 2. \times 10^{-6}$
PKS 1830-211	Ellingsen, Voronkov, Breen & Lovell (2012)	0.89	$(\leq 6.3 \times 10^{-7})$
B0218+357	Flambaum & Kozlov (2007)	0.6847	$(0.6 \pm 1.9) \times 10^{-6}$
B0218+357	Murphy et al. (2008)	0.6847	$\leq 1.8 \times 10^{-6}$
B0218+357	Kanekar (2011)	0.6847	$(\leq 3.6 \times 10^{-7})$
Milky Way	Levshakov, Agafonova, Molaro, & Reimers (2008)	0.0	$\leq 3 \times 10^{-8}$
Milky Way	Molaro et al. (2009)	0.0	$(4 - 14) \times 10^{-8}$
Milky Way	Levshakov et al. (2010)	0.0	$(26 \pm 3 \times 10^{-9})$
Milky Way	Levshakov, Kozlov & Reimers (2011)	0.0	$\leq 2.8 \times 10^{-8}$

**Table A1.** Recent Astronomical  $\Delta\mu/\mu$  Measurements